Imperial College London

Topic 8

Capacitor Circuits

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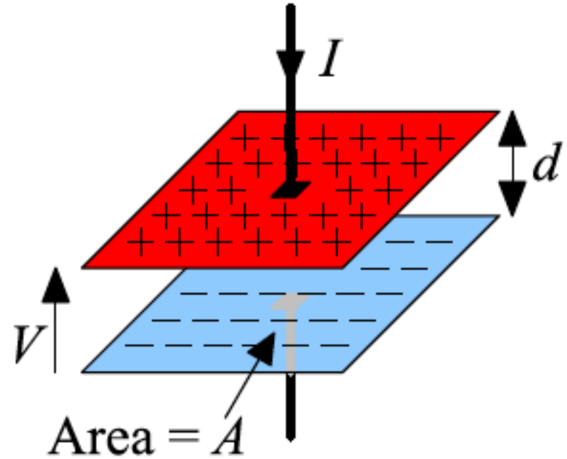
Capacitors & Capacitance

- A capacitor is formed from two conducting plates separated by a thin insulating layer called a **dielectric**.

- If a current i flows, positive change, q, will accumulate on the upper plate. To preserve charge neutrality, a balancing negative charge will be present on the lower plate.



- There will be a potential energy difference (or voltage v) between the plates proportional to q.

$$v = \frac{d}{A\epsilon} q$$

where A is the area of the plates, d is their separation and ε is the permittivity of the insulating layer ($\varepsilon_0 = 8.85\ pF/m$ for a vacuum).

- The quantity $C = A\varepsilon/d$ is the capacitance and is measured in Farads (F). Hence $q = Cv$, and the current i is the rate of charge on the plate.

The capacitor equations:

$$q = Cv, \ \ therefore \ \ \frac{dq}{dt} = i = C\frac{dv}{dt} \ \ and \ \ v = \frac{1}{C}\int i\, dt$$

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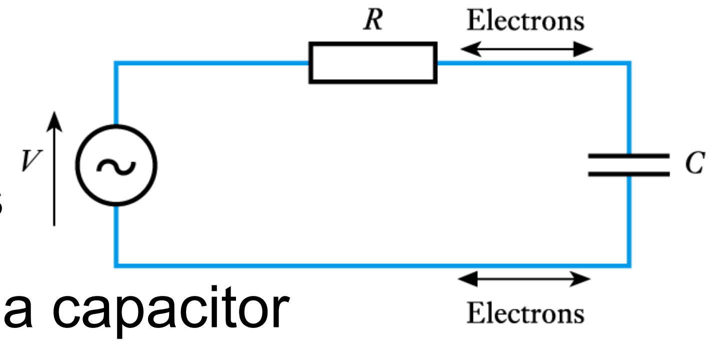
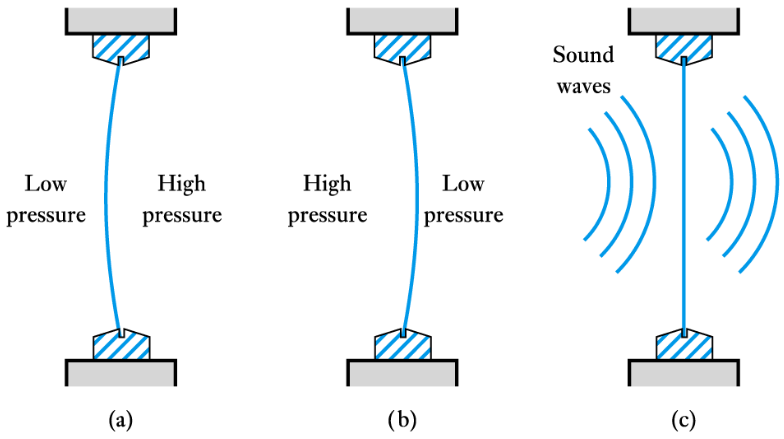
DC, AC and Capacitors

- A constant current (DC) cannot flow through a capacitor
  - There is an insulator between the two terminals

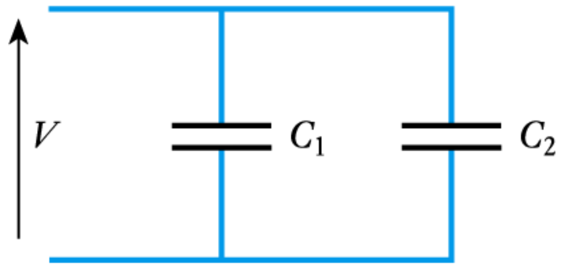


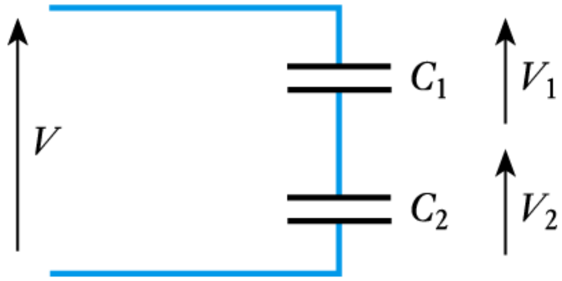
- An alternating current (AC) can "flow through" a capacitor
  - Since the voltage across a capacitor is proportional to the charge on it, an alternating voltage must correspond to an alternating charge
  - This can give the impression that an alternating current flows through the capacitor
- A mechanical analogy:
  - Air (charge) cannot pass through a window in spite of the pressure difference (voltage potential)
  - However, alternating pressure can make the window vibrates, produces air movement



Capacitors in Series and in Parallel

- **Capacitors in parallel**
  - consider a voltage V applied across two capacitors
  - then the charge on each is
    $Q_1 = VC_1$ *and* $Q_2 = VC_2$
  - if the two capacitors are replaced with a single capacitor C which has a similar effect as the pair, then
    Charge stored on combined C is $Q = Q_1 + Q_2$
    $$\Rightarrow VC = VC_1 + VC_2$$
    $$\Rightarrow C = C_1 + C_2$$





- **Capacitors in series**
  - consider a voltage V applied across two capacitors in series
  - the only charge that can be applied to the lower plate of C_1 is that supplied by the upper plate of C_2. Therefore the charge on each capacitor must be identical.
  - Let this be Q, and therefore if a single capacitor C has the same effect as the pair, then:
    $$V = V_1 + V_2 \Rightarrow Q/C = Q/C_1 + Q/C_2$$
    $$\Rightarrow 1/C = 1/C_1 + 1/C_2$$

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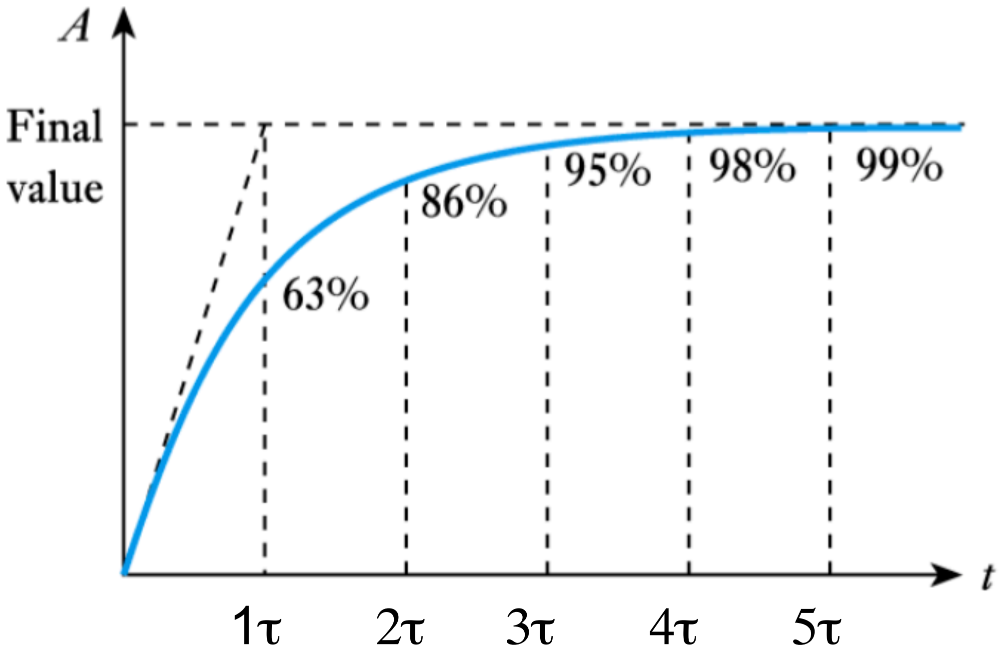
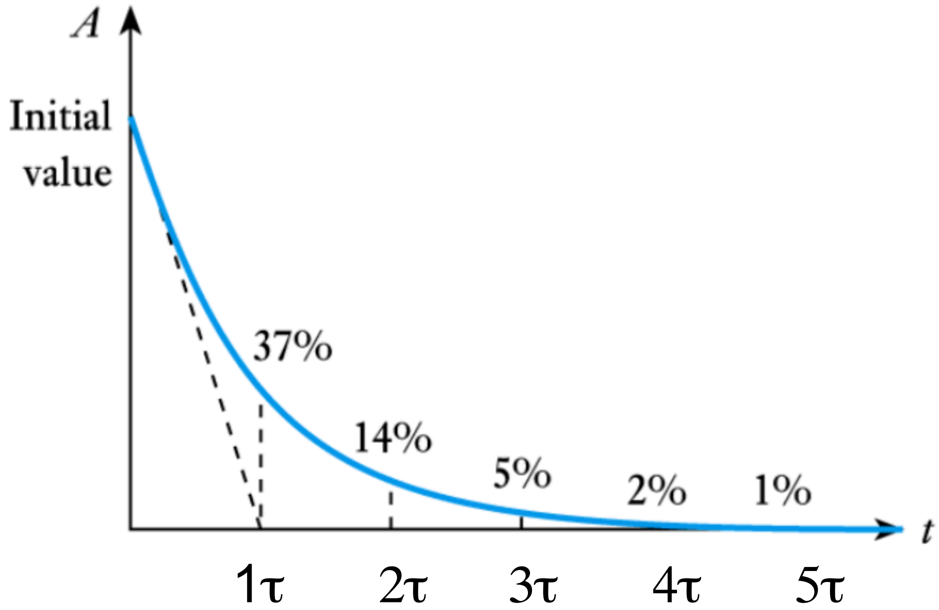
The Exponential Signal



$$y(t) = A(1 - e^{-t/\tau})$$



$$y(t) = Ae^{-t/\tau}$$

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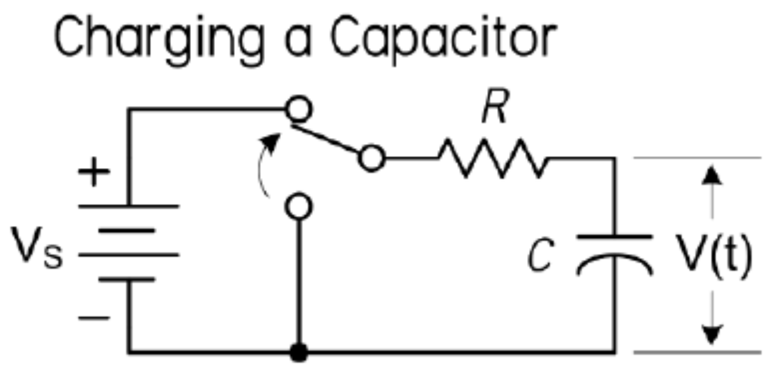
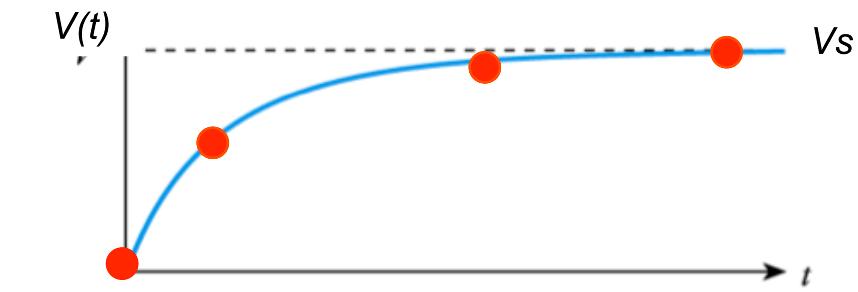
Capacitor and the Exponential

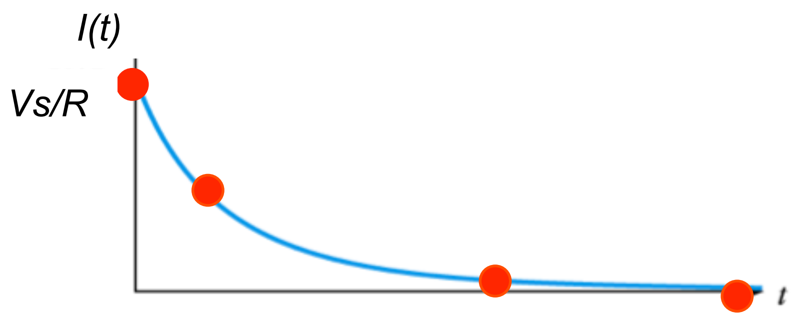
Consider the circuit shown here:

- Initially, the switch is in the down position. The input is connected to GND and the capacitor is discharged.
- At t = 0, the switch goes to up position. The battery voltage Vs is applied to the RC circuit and the capacitor starts charging.

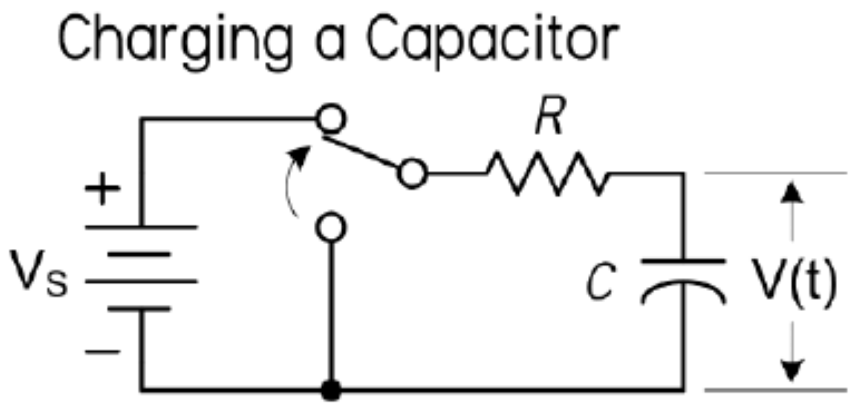


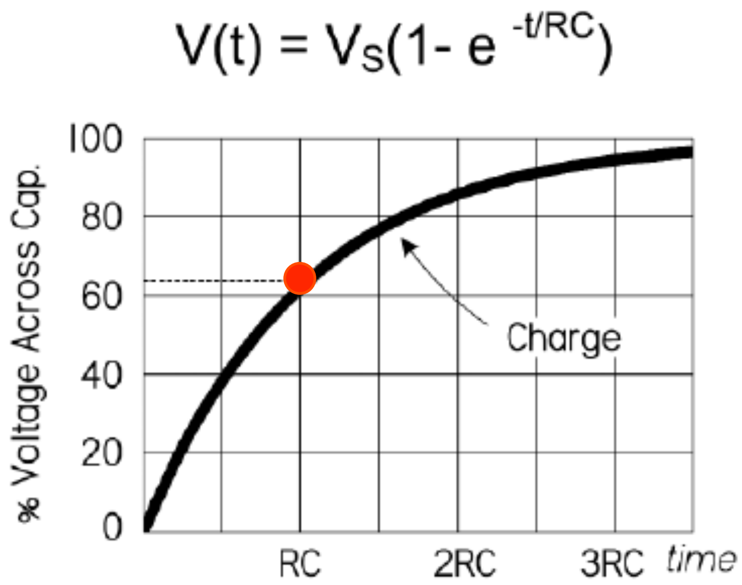
- At t = 0, V(t) is initially zero. The voltage across R is initially Vs. Therefore the charging current is Vs/R.
- As the capacitor charges:
  - ➢ *V(t)* increases
  - ➢ V_R (voltage cross the resistor) decreases
  - ➢ *I(t)* the charging current decreases
  - ➢ This result in the exponential behaviour of both *V(t)* and *I(t)*





RC circuit and time constant





- **Time constant**
  - Charging current *I* is determined by *R* and the voltage across it
  - Increasing *R* will increase the time taken to charge *C*
  - Increasing *C* will also increase time taken to charge *C*
  - Time required to charge to a particular voltage is determined by ***CR***
  - This product ***CR*** is the **time constant** τ (greek tau)

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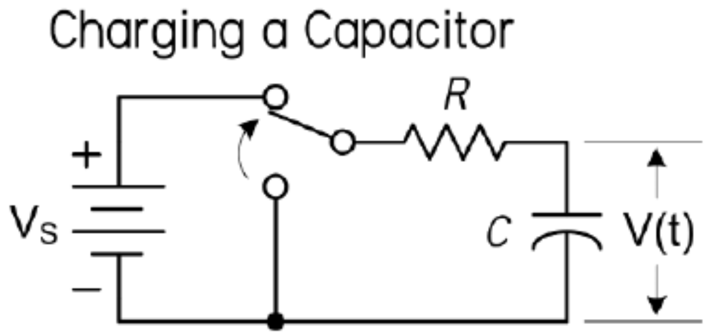
Step Response of a RC circuit

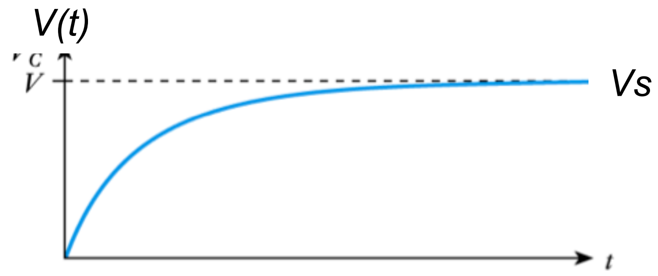
- Consider what happens to the circuit shown here as the switch is closed at t = 0.
- Apply KVL around the loop, we get:



$$iR + v = Vs, \quad but \quad i = C\frac{dv}{dt} \quad therefore \quad RC\frac{dv}{dt} + v = Vs$$

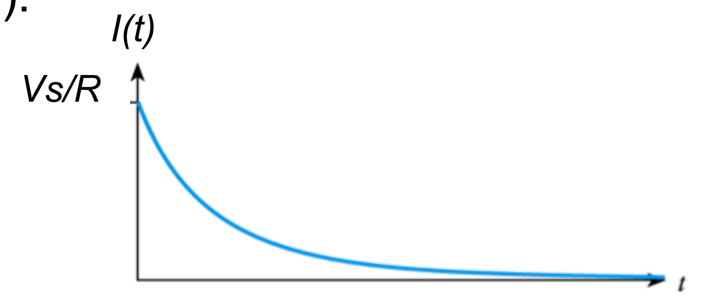
- This is a simple **first-order differential equation** with constant coefficients.
- Assuming $V(t)$ = 0 at t = 0, the solution to this is:

$$\boxed{V(t) = V_S(1 - e^{-t/RC})}$$



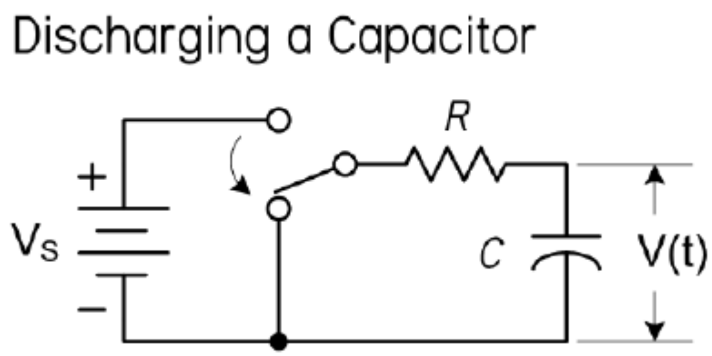
- Since $i = C\dfrac{dv}{dt}$ this gives (assuming V(t) = 0 at t = 0):

$$i = I \times e^{-\frac{t}{RC}} = I \times e^{-\frac{t}{\tau}}, \quad \text{where } I = \frac{Vs}{R}$$



Discharging Capacitor in a RC circuit

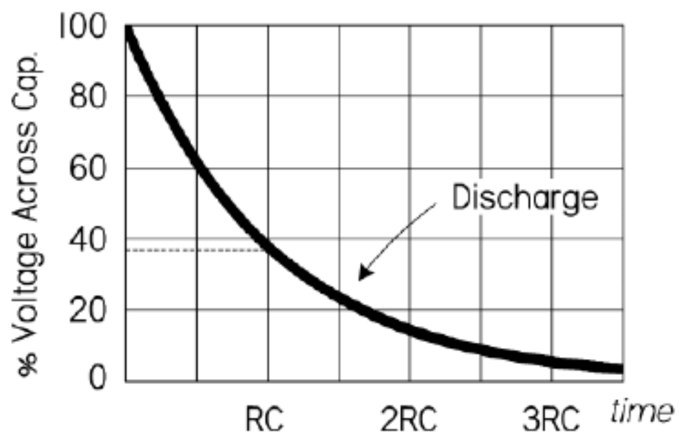
- Consider what happens to the circuit shown here as the left switch is open and the right switch closed at t = 0.
- At $t = 0$, $V(t) = V_s$.
- Apply KVL around the right loop, we get:



$$iR + v = 0, \quad and \quad i = C\frac{dv}{dt} \quad therefore \quad RC\frac{dv}{dt} + v = 0$$

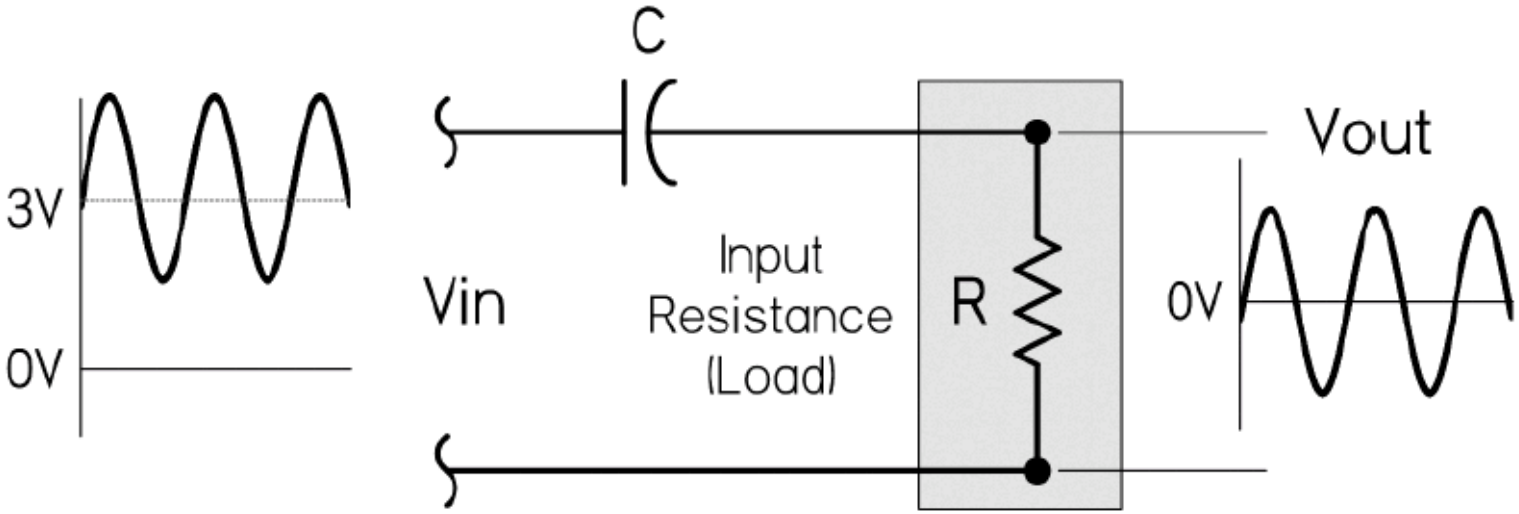
- Solving this simple first-order differential equation gives:

$$V(t) = V_S\, e^{-t/RC}$$



DC Blocking using Capacitor

- Capacitor is often used to present dc voltage from passing from one side of the circuit to another.
- Here, on the left side, the signal has a 3V DC component, and a sinewave superimpose.
- On the right side, the output signal Vout is centred around 0V. That is, the DC input is “blocked” or isolated from the output.
- This use of capacitor is also known as “**AC coupling**”.



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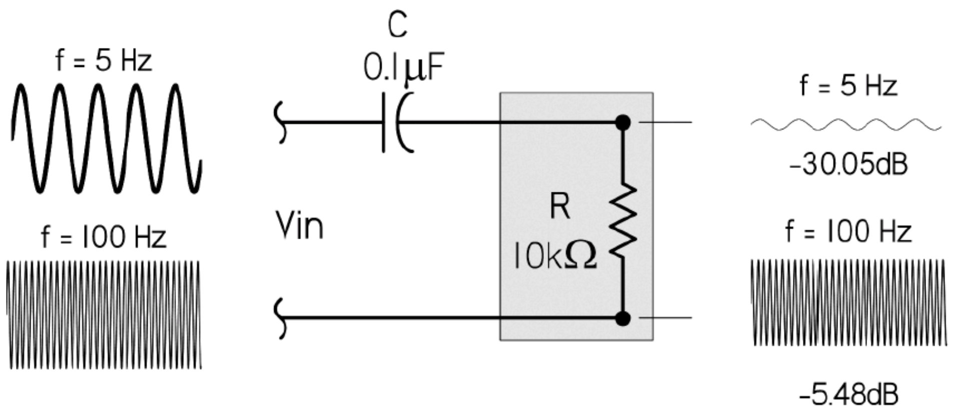
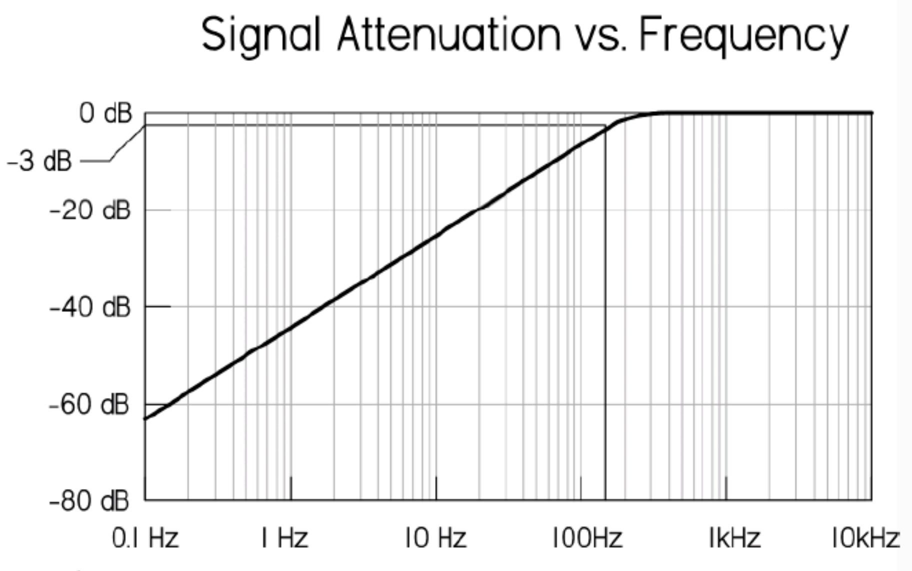
Filtering effect of Capacitor

- Such circuit also has different effect on the input signal at different frequencies.
- Shown here is two signals, one at 5Hz and another at 100Hz and the C and R values are as given.
- The 5Hz sinewave is suppressed by -30dB or reduced by a factor of 32.
- The 100Hz signal is only reduced by -5.48dB or reduced by a factor of 1.9.
- Therefore, a C in series with a R as shown will give us a high pass filter: a circuit that passes high frequency signals but suppresses low frequency.





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Decibel (dB)

- Ratio of output to input voltage in an electronic system is called voltage **gain**:

$$A = \left| \frac{V_{out}}{V_{in}} \right|$$

- If the gain is low than 1, we also call this attenuation.
- Voltage gain of a circuit is often expressed in logarithmic form:

$$A\,(in\ \ dB) = 20\ \log_{10} \left| \frac{V_{out}}{V_{in}} \right|$$

- Power gain of a circuit is the ratio of output power to input power, and is also often expressed in dB, but the equation is different:

$$Power\_Gain\,(in\ \ dB) = 10\ \log_{10} \left| \frac{P_{out}}{P_{in}} \right|$$

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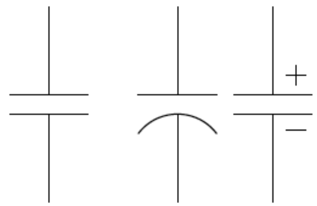
Types of Capacitors

- Capacitor symbol represents the two separated plates. Capacitor types are distinguished by the material used as the insulator.



- **Polystyrene**: Two sheets of foil separated by a thin plastic film and rolled up to save space. Values: 10 pF to 1 nF.

- **Ceramic**: Alternate layers of metal and ceramic (a few µm thick). Values: 1 nF to 1 µF.

- **Electrolytic**: Two sheets of aluminium foil separated by paper soaked in conducting electrolyte. The insulator is a thin oxide layer on one of the foils. Values: 1 µF to 10mF.

- Electrolytic capacitors are **polarised**: the foil with the oxide layer must always be at a positive **voltage** relative to the other (else **explosion**).
- Negative terminal indicated by a curved plate in symbol or "-".

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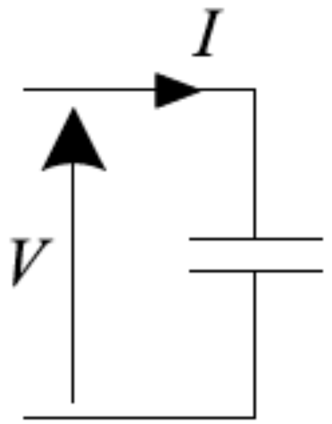
Current / Voltage Continuity

Capacitor: $i = C\, dv / dt$

- For the voltage to change abruptly $dv / dt = \infty \Rightarrow i = \infty$.

This never happens so ...

- **The voltage across a capacitor never changes instantaneously.**
- Informal version: A capacitor "tries" to keep its voltage constant.



Summary

- Capacitor:
  - $i = C\, dv/dt$
  - parallel capacitors add in value
  - v across a capacitor never changes instantaneously
  - When charging a capacitor with a constant DC voltage through a resistor, the capacitor voltage rises exponentially
  - The time constant of the exponential is the product of R and C.